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Question 1

Find the point(s) on the parabola $y = x^3 - 6x + 1$ where the tangent is parallel to the line $6x - y + 11 = 0.$ Also, determine the equation(s) of the tangent at that point(s).

Let $f(x) = x^3 - 6x + 1$

The derivative is

$f'(x) = 3x^{3-1} - 6x^{1-1} + 0$

$f'(x) = 3x^2 - 6$

The line $6x - y + 11 = 0$ can be expressed as $y = 6x + 11$ so its slope is 6

The slope of the tangent line parallel to $6x - y + 11 = 0$ is also 6.

We must solve $f'(x) = 0$ for x

We have

$3x^2 - 6 = 6$

$3x^2 = 12$

$x^2 = 4$

$x = 2$ or $x = -2$

The points of tangency are $(2, f(2))$ and $(-2, f(-2))$ which correspond to

$(2, -3)$ and $(-2, 5)$

The equation of tangent lines with slope of 6 are

$y - (-3) = 6(x - 2)$ or $y = 6x - 15$

and

$y - 5 = 6(x - (-2))$ or $y = 6x + 17$

Final answers:

Points of tangency: $(2, -3)$ and $(-2, 5)$

Tangent lines: $y = 6x - 15$ and $y = 6x + 17$

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Question 2

Find constants a, b, c, and d such that the curve $y = ax^3 + bx^2 + cx + d$ has horizontal lines at the points (0, 1) and (1, 0) on the curve.

Solution:

Let $f(x) = ax^3 + bx^2 + cx + d$

The slope of the horizontal tangent lines at the points (0, 1) and (1, 0) is zero

$f(0) = d = 1$ because the curve contains the point (0, 1)

$f(1) = a + b + c + d = 0$ because the curve contains the point (1, 0)

$a + b + c = -1$ because $d = 1$

The derivative is

$f'(x) = 3ax^2 + 2bx + c$

$f'(0) = c = 0$ because the slope of horizontal tangent line is zero

$a + b = -1$ because $c = 0$

$f'(1) = 3a + 2b + 0 = 0$

Since $a + b = -1$, then $a = -b - 1$ and $3a + 2b + 0 = 0$ becomes

$3(-b - 1) + 2b = 0$

$b = -3$ and $a = -(-3) - 1 = 2$

Answer: $a = 2$, $b = -3$, $c = 0$, and $d = 1$

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Question 3

Find the following derivatives.
Give your answers in simplified factored form.

a) $\dfrac{(x^2+5)^3}{(2x^3-3)^2}$

Part-a

Using Quotient Rule and Chain Rule,

$$\frac{(2x^3-3)^2[3(x^2+5)^2(2x)]-(x^2+5)^3[2(2x^3-3)(6x^2)]}{[(2x^3-3)^2]^2}$$

$$=\frac{6x(x^2+5)^2(2x^3-3)^2-12x^2(2x^3-3)(x^2+5)^3}{(2x^3-3)^4}$$

$$=\frac{6x(x^2+5)^2(2x^3-3)[(2x^3-3)-2x(x^2+5)]}{(2x^3-3)^4}$$

$$=\frac{6x(x^2+5)^2(2x^3-3)(-3-10x)}{(2x^3-3)^4}$$

$$=\frac{-6x(x^2+5)^2(10x+3)}{(2x^3-3)^3}$$

Part-b

b) $f(x) = -x^2(3x-2)^2(x^3+2x+1)^3$

The function can be expressed as

$f(x) = -(3x^2-2x)^2(x^3+2x+1)^3$

Using product rule and Chain rule, the derivative is

$f\,'(x) = -(3x^2 - 2x)^2\,[3(x^3 + 2x + 1)^2(3x^2 + 2)] + (x^3 + 2x + 1)^3[-2(3x^2 - 2x)(6x - 2)]$

$= -x^2(3x - 2)^2\,[3(x^3 + 2x + 1)^2(3x^2 + 2)] + (x^3 + 2x + 1)^3[-2x(3x - 2)(6x - 2)]$

$= -x(3x - 2)(x^3 + 2x + 1)^2\{x(3x - 2)\,[3(3x^2 + 2)] + (x^3 + 2x + 1)[2(6x - 2)]\}$

$= -x(3x - 2)(x^3 + 2x + 1)^2\{\,(27x^4 - 18x^3 + 18x^2 - 12x) + (12x^4 - 4x^3 + 24x^2 + 4x - 4)\}$

$= -x(3x - 2)(x^3 + 2x + 1)^2(39x^4 - 22x^3 + 42x^2 - 8x - 4)$